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# Laminar free-convective heat transfer from a vertical isothermal plate to water at low temperatures with variable physical properties

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**Abstract**

All studies concerning laminar free convection along a vertical isothermal plate in water at low temperatures have been conducted assuming constant dynamic viscosity and thermal conductivity both taken at ambient or film temperature. In this study the problem has been treated taking into account the temperature dependence of all water physical properties. The results are obtained with the numerical solution of the boundary layer equations. The variation of $\mu$ and $k$ with temperature has a small influence on wall heat transfer but a strong influence on wall shear stress. These quantities show a significant reduction at density extremum. 

*Keywords:* Heated vertical plate; Free convection; Water; Similarity; Density extremum

## 1. Introduction

A characteristic common to most analytical studies of natural convection has been the neglect of all fluid-property variations, except for the essential density difference, which, in the absence of mass transfer, are a consequence of temperature gradients in the fluid. This greatly simplifies analytical studies since the number of variables which must be considered is vastly reduced.

The classical problem of free convection heat transfer from a heated vertical isothermal surface has been mainly treated assuming constant viscosity and thermal conductivity and a linear density–temperature relationship. An important method for finding a solution to this problem is the similarity method (Schuh, 1948; Ostrach, 1953).

There are some works concerning water free convection in low temperature range where the density–temperature relationship is nonlinear. Goren (1966) solved the problem of water free convection along a vertical isothermal surface with ambient temperature equal to maximum density temperature. The obtained results are valid for surface temperatures until 8°C. Vanier and Tien (1967) extended the above work to surface temperatures until 35°C but the results were obtained again only for one ambient temperature equal to 4°C. Gebhart and Mollendorf (1978) analyzed the problem of laminar free convection of water over a vertical plate with both thermal and saline diffusion and presented results for the vertical isothermal plate with temperature diffusion. In a recent paper, Pantokratoras (1999) showed that similarity exists at every ambient temperature, even in the nonlinear region, and presented results for the isothermal plate in the entire temperature range between 20 and 0°C for pure and saline water. However, all the above-mentioned works concerning water assume constant dynamic viscosity and thermal conductivity both taken at ambient or film temperature.

There are two excellent works which contain literature review for natural convection flows with variable fluid properties (Carey and Mollendorf, 1980; Kakac et al., 1985). Most of the works cited in the above two papers concern air, gases and oils. It appears that the only work available which concerns water free convection along a vertical isothermal plate, taking into account the temperature dependence of all physical properties, is that of Nishikawa and Ito (1969). However, this work concerns water at ambient temperatures between 370.82 and 387.98°C and supercritical pressures. The objective of the present paper is to present results for laminar free convection of water along a vertical isothermal plate in the temperature range between 20 and 0°C, taking into account the temperature dependence of viscosity and thermal conductivity as well as the nonlinearity between density and temperature.

## 2. The mathematical model

Consider laminar free convection along a vertical isothermal plate placed in a calm environment with $u$ and $v$ denoting, respectively, the velocity components in the $x$ and $y$ direction, where $x$ is vertically upwards and $y$ is the coordinate perpendicular to $x$. For steady, two-dimensional flow in the absence of viscous dissipation, motion pressure and volumetric energy

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source effects, the boundary layer equations appropriate to the variable-property situation are (Kakac et al., 1985)

*Continuity equation*:

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0; \tag{1}$$

*Momentum equation*:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\rho-\rho_a}{\rho_a}g; \tag{2}$$

*Energy equation*:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right); \tag{3}$$

where $T$ is the water temperature, $\rho$ and $\rho_a$ are the local and ambient water density, $\mu$ is the dynamic viscosity, $k$ is the thermal conductivity and $c_p$ is the specific heat. The density of saline water is a function of temperature, salinity and pressure. In this paper the international equation of state for seawater, known from oceanography (Fofonoff, 1985), is used for the calculation of density. The dynamic viscosity, thermal conductivity and specific heat have been calculated using data given by Kukulka et al. (1987).

The following boundary conditions were applied:

$$\text{at } y=0: \quad u=v=0, \quad T=T_o,$$
$$\text{as } y\to\infty: \quad u=0, \quad T=T_a.$$

Eqs. (1)–(3) form a parabolic system and are solved by a method described by Patankar (1980). The finite difference method is used with primitive coordinates $x$, $y$ and a space marching procedure is used in the $x$ direction with an expanding grid.

## 3. Results and discussion

The most important quantities of this problem are the wall heat transfer and the wall shear stress given by the following equations:

$$\varphi'(0)=\frac{x}{T_o-T_a}\left[\frac{Gr_x}{4}\right]^{-1/4}\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{4}$$

$$f''(0)=\frac{\rho_o x^2}{\mu_o\sqrt{2}}[Gr_x]^{-3/4}\left(\frac{\partial u}{\partial y}\right)_{y=0}. \tag{5}$$

$Gr_x$ is the local Grashof number defined as

$$Gr_x=\frac{g\rho_f^2x^3}{\mu_f^2}\frac{\rho_a-\rho_o}{\rho_a}, \tag{6}$$

where $\rho_a$, $\rho_o$ are the ambient and wall local density, $\rho_f$ and $\mu_f$ are the film density and film dynamic viscosity both calculated at film temperature $(T_o+T_a)/2$.

In the present work, at each downstream position, the velocity and temperature profiles have been calculated from the upstream profiles taking into account the temperature dependence of $\mu$, $k$ and $\rho$ across the boundary layer. The temperature and velocity derivatives in Eqs. (4) and (5) were calculated from the corresponding known downstream profiles. The accuracy of the method was tested comparing the results with those of the classical free convection problem with constant viscosity and thermal conductivity and linear relationship between density and temperature (Pantokratoras, 1999).

It should be noted here that when viscosity and thermal conductivity are constant the problem admits similarity solution and in that way has been treated until now. But does similarity exist for this problem with all temperature-dependent variables, that is, viscosity and thermal conductivity functions of temperature and nonlinear density–temperature relationship? This question has not been answered in the literature until now.

In the present work this problem has been treated as follows. The nondimensional velocity $f'$ and temperature $\varphi$ profiles have been calculated along the plate for different ambient temperatures. In Fig. 1 three groups (1, 2, 3) of nondimensional velocity profiles are shown. In each group three curves, calculated at three different downstream positions from the plate edge, are included. The plate temperature was $T_o=25°C$ and the ambient water temperature was $T_a=10°C$. The first group of velocity profiles has been produced with $\mu$, $k$ and $\rho$ variable across the boundary layer. The second group of velocity profiles has been produced with $\rho$ variable across the boundary layer and $\mu$ and $k$ constant, calculated at film temperature. In the third group $\rho$ is variable across the boundary layer and $\mu$ and $k$ constant, calculated at ambient temperature. In each group the shape of the three velocity profiles is the same and the profiles are very close. The differences are below 1%. The coincidence of curves in the second and third group is reasonable because for these situations similarity exists. But why are the three curves of the first group identical too? It could be claimed that similarity exists also for this case, that is, when $\mu$, $k$ and $\rho$ are functions of temperature, but for this conclusion an analytical proof is needed. The same things are valid also for the temperature profiles shown in Fig. 2. From a practical point of view, there is one velocity gradient $(\partial u/\partial y)_{y=0}$ for the three velocity curves and one temperature gradient $(\partial T/\partial y)_{y=0}$ for the three temperature curves produced with $\mu$, $k$ and $\rho$ variable across the boundary layer (group 1) and these values have been used in the present work, although no analytical proof is given for similarity.

It is important to note that the real water velocity profile ($\mu$ and $k$ variable across the boundary layer, first group of curves) has the greatest inclination to the plate, that is, the greatest wall shear stress. From Fig. 1 it is clear that the results presented until now in the literature have underestimated the real water wall shear stress. The above-described method, which has been used successfully for the laminar free convection from a vertical isothermal plate to water at 4°C (Pantokratoras, 2001), has been extended in the present work to the entire temperature range between 20 and 0°C.

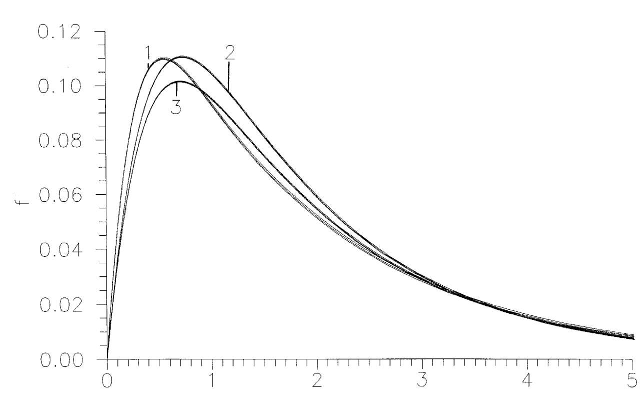


Fig. 1. Nondimensional velocity profiles at three different downstream positions from the plate edge ($x=80$, 100, 120 cm) for $T_o=25°C$ and $T_a=10°C$: 1 – $\mu$ and $k$ variable across the boundary layer; 2 – $\mu$ and $k$ constant at film temperature; 3 – $\mu$ and $k$ constant at ambient temperature ($\rho$ temperature dependent, variable across the boundary layer for all cases).

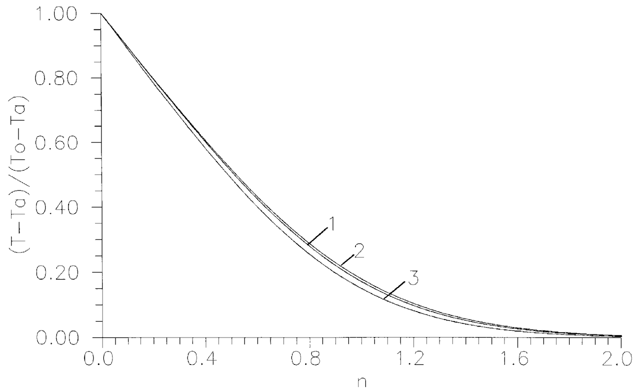


Fig. 2. Nondimensional temperature profiles at three different downstream positions from the plate edge ($x = 80$, 100, 120 cm) for $T_o = 25°C$ and $T_a = 10°C$: 1 – $\mu$ and $k$ variable across the boundary layer; 2 – $\mu$ and $k$ constant at film temperature; 3 – $\mu$ and $k$ constant at ambient temperature ($\rho$ temperature dependent, variable across the boundary layer for all cases).

As was mentioned before the results produced in this work take into account both the nonlinear relationship between density and temperature of water and the variation of viscosity and thermal conductivity with temperature. As is expected the results depend on the temperature difference between the plate and the ambient water. For a given ambient water temperature and a constant $Pr_a$ calculated at this temperature, results were produced for four $\Delta T$ (1, 5, 10, 15°C) in the temperature range between 20 and 3.98°C and for one $\Delta T$ (1°C) in the temperature range between 3.98 and 0°C. The correspondence between the ambient Prandtl number and the ambient water temperature is as follows:

| $T_a$ | 20.00 | 15.00 | 10.00 | 5.00 | 3.98 | 3.00 | 2.00 | 1.00 |
|---|---|---|---|---|---|---|---|---|
| $Pr_a$ | 6.99 | 8.01 | 9.31 | 10.99 | 11.40 | 11.80 | 12.24 | 12.70 |

In Figs. 3 and 4 the wall heat transfer $-\varphi'(0)$ and the wall shear stress $f''(0)$ are shown as functions of $Pr_a$ for different temperature differences. From Fig. 3 it is seen that for every $Pr_a$ the wall heat transfer decreases as the temperature difference increases. Another interesting conclusion drawn from this figure is that, for the first three $Pr_a$ (6.99, 8.01, 9.31), the heat transfer increases with $Pr_a$ increase, whereas from $Pr_a = 9.31$ to 11.40 the heat transfer decreases as the $Pr_a$ increases. In the range below 3.98°C ($Pr_a = 11.40$) the heat transfer increases as $Pr_a$ increases following the same trend that is valid in the range between $Pr_a = 6.99$ and 9.31. From Fig. 4 it is seen that, as $Pr_a$ increases and the temperature difference decreases, the wall shear stress decreases. In the temperature range below 3.98°C the wall shear stress increases as $Pr_a$ increases.

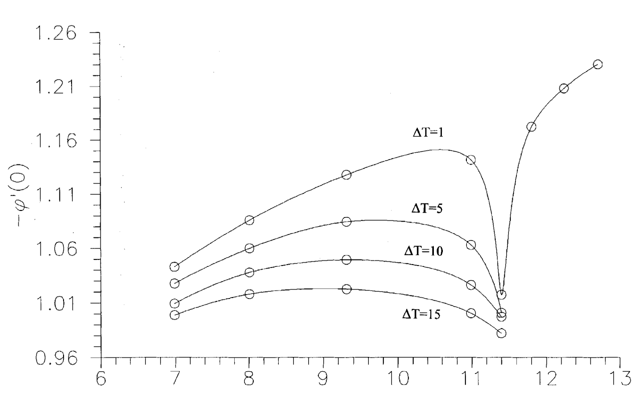


Fig. 3. Wall heat transfer as function of $Pr_a$ and different temperature differences.

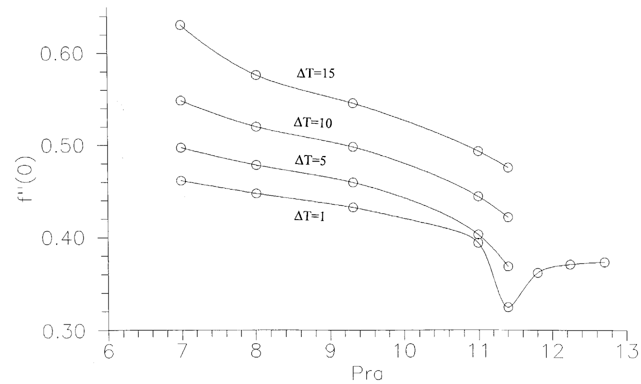


Fig. 4. Wall shear stress as function of $Pr_a$ and different temperature differences.

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